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Modelling Day-Ahead and Intraday Electricity Markets

Ökonomik der Energiewende , KIT

Rüdiger Kiesel | Chair for Energy Trading and Finance | University of Duisburg-Essen

Part I

Electricity Trading

Electricity Markets

Traded Products

Short-term Markets

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Agenda

Electricity Markets

Traded Products

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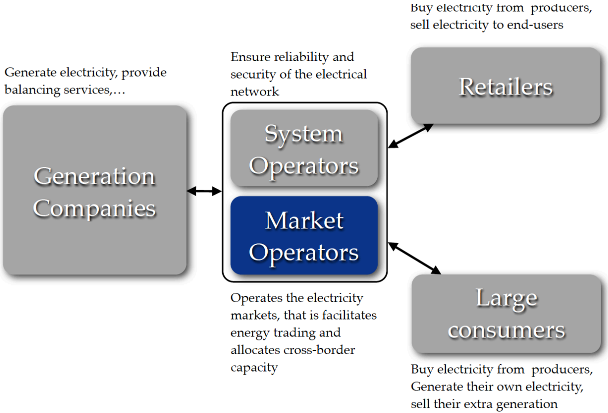
Organisation of the Power System



Types of Electricity Markets

A centralized platform where participants can exchange electricity transparently according to the price they are willing to pay or receive, and according to the capacity of the electrical network.

- Fixed Gate Auction
  - Participants submit sell or buy orders for several areas, several hours,
  - the submissions are closed at a pre-specified time (closure)
  - the market is cleared.
  - Example: day-ahead.
  - (Intraday has a fixed gate auction at 15:00)
- Continuous-time Auction
  - Participants continuously submit orders. Orders are stored,
  - Each time a deal is feasible, it is executed,
  - Example: intraday.

Electricity Exchanges

Electricity related contracts can be traded at exchanges such as

- the Nord Pool, mainly Northern European countries, `http://www.nordpoolspot.com/`
- the European Energy Exchange (EEX), `http://www.eex.com/en`
- EPEX, located in Paris, founded by EEX and Powernext (French Energy Exchange); Electricity spot market for Germany, Austria, France and Switzerland; `http://www.epexspot.com/en/`
- Amsterdam Power Exchange (APX), covers the Netherlands, Belgium and the UK, `http://www.apxgroup.com`

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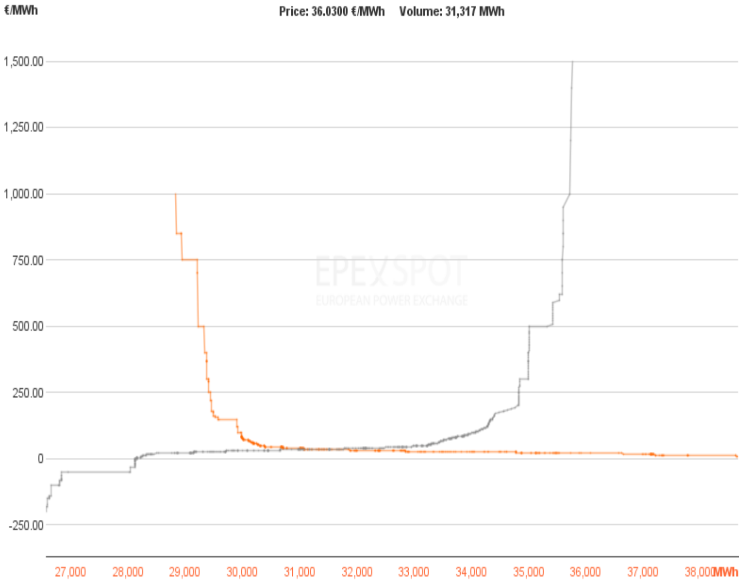
EEX – traded products

- Futures contracts for Germany and France with delivery periods week, month, quarter, year.
- For Germany single days and weekends are available.
- European style options on futures.

EPEX – traded products

- Auction day-ahead and continuous intra-day market.
- Products are individual hours, baseload, peakload, blocks of contiguous hours.
- Intraday market is open 24 hours a day, 7 days a week and products can be traded until 45 minutes before delivery.
- For Germany 15 minutes contracts can be traded.

Auction EPEX



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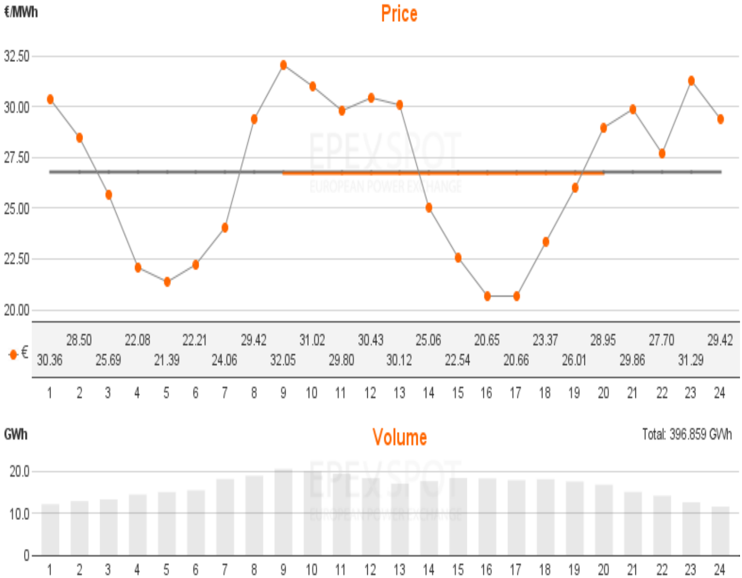
Daily spot price EPEX



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Electricity Markets

Traded Products

Short-term Markets

Day-Ahead Market

- Possibility to correct long-term production schedule (build on the forward markets) in terms of hourly production schedule of power plants (Delta Hedging) – sell more expensive hours, buy cheaper hours for flexible power plants.
- Adjust for residual load profiles on an hourly basis
- Market for production from renewable energy sources (wind, solar) as on long-term markets only averages can be traded

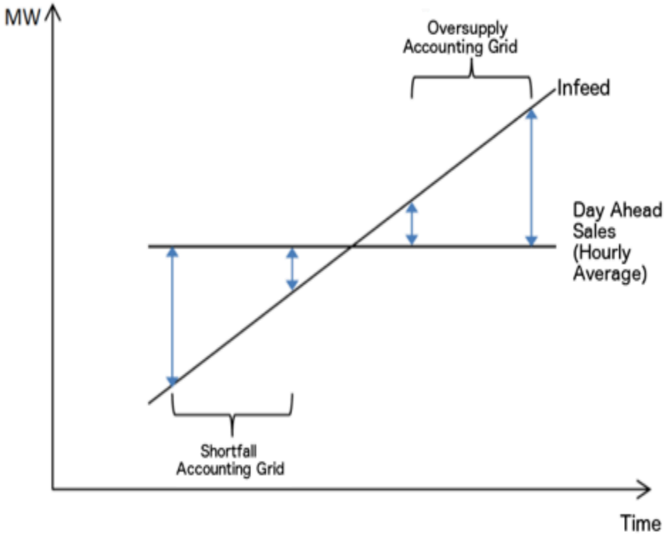
Intraday Market

- Trading of hours, quarter-hours until 45 min before start of period continuously during the day
- From 15:00 hours of next day, from 16:00 quarter-hours of next day

Motivation for Trading Intra-Day Market

- Correction or optimisation of Day-Ahead position
  - power plant outages
  - optimisation of power plant usage (generator)
  - optimisation of demand (costumer)
  - renewable energy producer – changes of forecasts
- Balancing quarter hour-ramps with quarter-hour contracts
- Proprietary trading

Quarter Hour Ramps



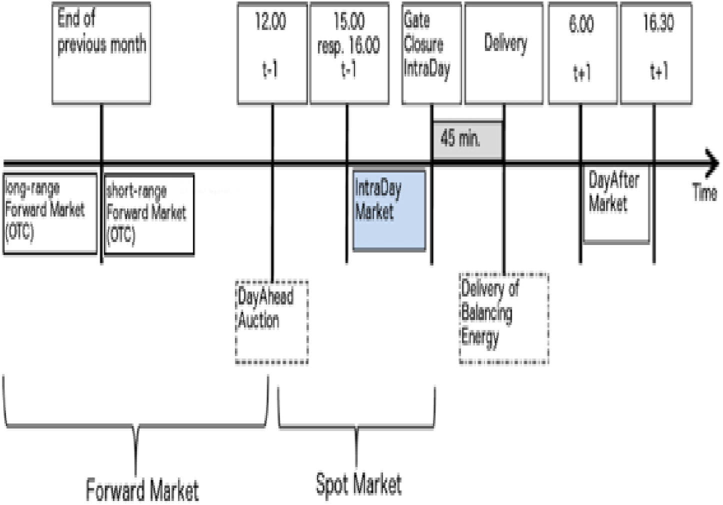
Structure of Balancing and Reserve Markets

In Europe, the *European Network of Transmission System Operators for Electricity,(ENTSO-E)* coordinates overarching grid topics. The main task of a TSO is to ensure a constant power frequency in the transmission system. The following control actions are applied

- *Primary Reserve* starts within seconds as a joined action of all TSOs in the system.
- *Secondary Reserve* replaces the primary reserve after a few minutes and is put into action by the responsible TSOs only.
- *Tertiary Reserve* frees secondary reserves by rescheduling generation by the responsible TSOs.

The TSO tenders the required products for fulfilling these functions. Reserve products may involve payments for the availability of the reserved capacity.

Timing Electricity Trading



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Part II

Intraday Trading

Intraday Trading

Model Architecture

- Data
- Model Variables
- Threshold Model
- Intra-day Last vs DayAhead
- Intra-day Continuous

Results

- DayAhead vs Intra-day Last
- Price Delta Continuous Trading

Agenda

Intraday Trading

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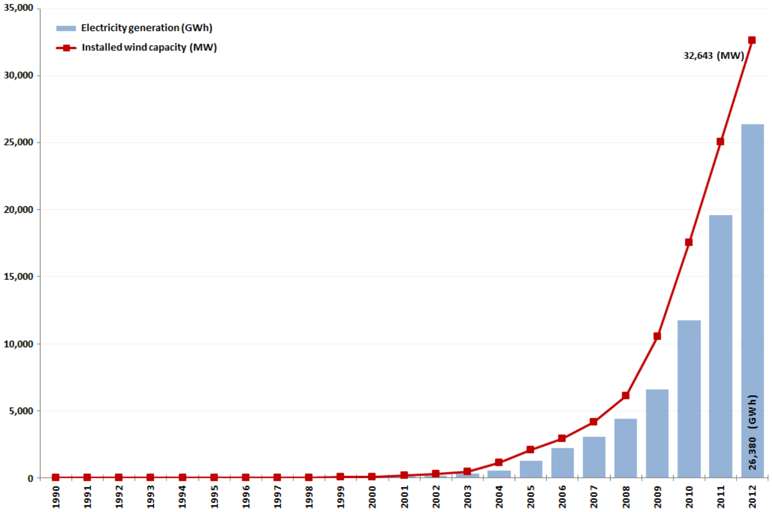
Increase of Renewables: Photovoltaik



Source: Federal Ministry for the Environment, Nature Conservation and Nuclear Safety, 2013.

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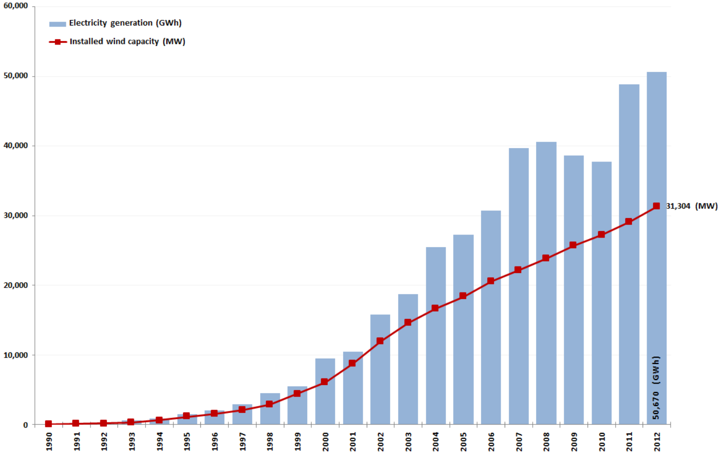
Increase of Renewables: Wind



Source: Federal Ministry for the Environment, Nature Conservation and Nuclear Safety, 2013.

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Price Effect of Renewables

- An investigation in the merit-order effect is given by [**Cludius2014**].
- They find that electricity generation by wind and PV has reduced spot market prices by 6 €/MWh in 2010 rising to 10 €/MWh in 2012.
- They also show that merit order effects are projected to reach 14-16 €/MWh in 2016.

Historical Data Basis

- In this section we use data from the EPEX from 01.01.2009 until 30.11.2013.
- This are 1795 days, resp. 43080 single hours.
- We have 82 hours with no Intraday trading for which we use the corresponding Day-ahead prices.
- As an Intraday hour is traded up to 33 hours we report the last price and the volume-weighted average price.
- We only use single hour trading periods (no blocks, no quarter hours).

Volume DayAhead and IntraDay

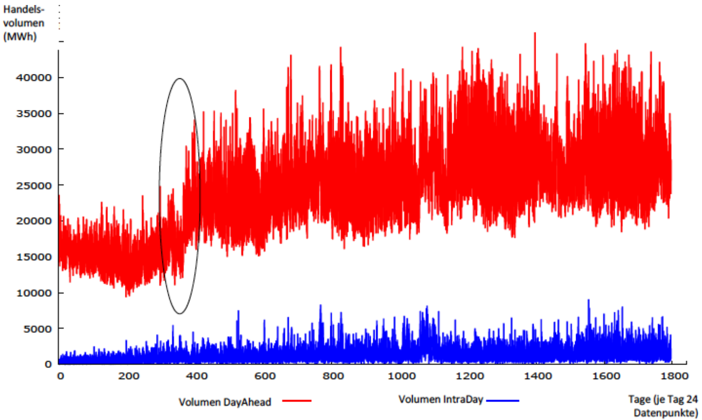

Figure: Volume DayAhead and IntraDay, hourly

Volume Discussion

- Day-ahead shows higher volume.
- One can observe the volume jump with the introduction of direct marketing of renewable energy.
- Day-ahead and Intraday are trending upwards, which can be explained by the expansion of renewables in Germany.

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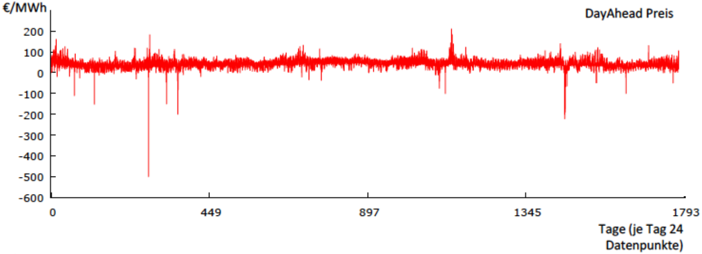
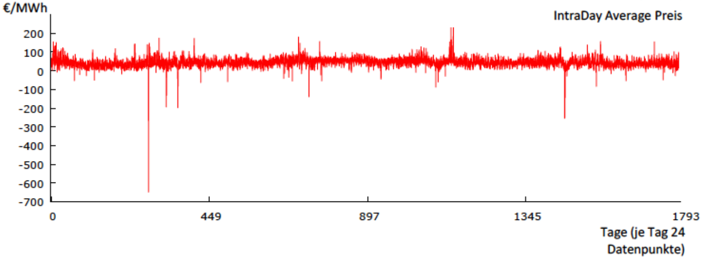
Day-Ahead vs Average IntraDay





Prices Summary Statistics

		2009	2010	2011	2012	2013	Gesamt
Mittelwert	DayAhead	38,85	44,49	51,12	42,69	37,97	43,09
	IntraDay Aver	39,01	45,62	51,22	43,87	38,76	43,76
	IntraDay Last	38,37	46,03	50,42	43,97	38,43	43,51
Standard-abweichung	DayAhead	19,41	13,97	13,60	18,60	15,73	17,12
	IntraDay Aver	24,93	16,56	15,48	19,34	16,75	19,52
	IntraDay Last	35,39	21,46	21,58	24,04	21,64	25,88
Minimum	DayAhead	-500,02	-20,45	-36,82	-221,99	-100,03	-500,02
	IntraDay Aver	-648,62	-62,62	-139,07	-254,09	-83,25	-648,62
	IntraDay Last	-1499,00	-190,00	-200,00	-320,00	-200,00	-1499,00
Maximum	DayAhead	182,05	131,79	117,49	210,00	130,27	210,00
	IntraDay Aver	173,72	180,07	156,22	265,30	155,61	265,30
	IntraDay Last	300,00	805,00	300,00	400,00	380,00	805,00
Median	DayAhead	38,07	45,09	51,85	42,12	36,50	42,97
	IntraDay Aver	37,86	45,08	52,19	43,35	37,30	43,48
	IntraDay Last	37,00	45,00	51,00	42,50	36,00	42,50
Kurtosis	DayAhead	80,96	1,52	1,86	42,94	2,39	40,96
	IntraDay Aver	139,21	3,62	9,24	26,76	2,41	84,66
	IntraDay Last	581,04	192,01	20,02	35,96	22,92	448,54
Schiefe	DayAhead	-3,23	-0,07	-0,64	-2,67	0,10	-1,84
	IntraDay Aver	-6,33	0,16	-1,24	-0,41	0,34	-2,98
	IntraDay Last	-16,65	5,59	-1,13	0,38	1,64	-8,05

Price Discussion

- Delta (difference) of the average values is positive in most cases – so there seems to be a premium for the short-term trading.
- There is a much higher volatility in the Intraday prices.
- 2012 shows significant differences in many classification values – this may be due to the lower volume, but also to the entrance of new players as direct-marketing started.

Correlation Prices Day-Ahead IntraDay

	2009	2010	2011	2012	2013
DayAhead - IntraDay Average	0,8598	0,8582	0,8462	0,8808	0,8389
DayAhead - Intraday Last	0,6570	0,6700	0,6544	0,6507	0,6479
Intraday Average - IntraDay Last	0,8470	0,8389	0,8316	0,8448	0,8608

Empirical Facts on Intraday Trading – Summary

- Dayahead shows higher volume – roughly 30 GW compared to 2 GW Intra-day (year 2013)
- One can observe the volume jump with the introduction of direct marketing of renewable energy in 2011.
- Dayahead and Intraday volume are trending upwards, which can be explained by the expansion of renewables in Germany.
- Price volatility Intraday is much higher than Dayahead.

Empirical Investigation

- Which factors explain differences between last Intraday prices and Day-ahead prices?
- Which factors explain Intraday price changes?
- Are there different regimes in which Intraday prices behave differently?
- Is there a time of the day effect in continuous trading? And different regimes?

Factors Influencing Prices

[**Hagemann2013**] and [**HagemannWeber2013**] construct a fundamental model and use 2010-11 data and analyse the difference of Day-ahead and volume-weighted Intraday. They find

- Ex-post forecast error wind significant. Here ex-post means the day-ahead forecast compared with the actual wind generation.
- Ex-post forecast error PV significant.
- Unplanned outages of power plants significant.
- Foreign demand/supply insignificant.
- There may be a portfolio netting within power plant portfolios.

Factors – Previous Literature

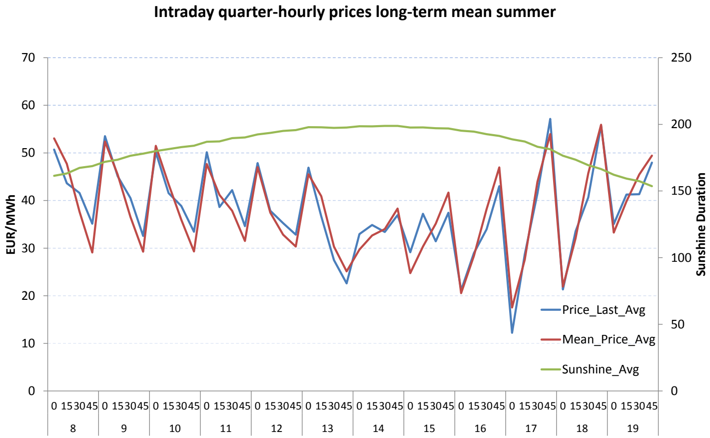
- Several studies have discussed the effects of prognosis errors for wind generation (see [**Ketterer2014**] and [**Nicolosi2010**]).
- Relation between demand and conventional power plants:
  - In [**JonssonPinsonMadsen2010**] it is shown that the ratio between wind and conventional power production affects the electricity price most (the so-called wind penetration).
  - [**NicolosiFuersch2009**] identify the residual load, the electricity demand that needs to be met by conventional power, as an important variable.
  - [**HagemannWeber2013**] show that the load supply ratio is of importance.
- To include trading volume as a fundamental variable is also supported by the literature as e.g. [**GraeberKleine2013**] find that the forecast for balancing costs in Intraday trading are linked to the trading volume.

Agenda

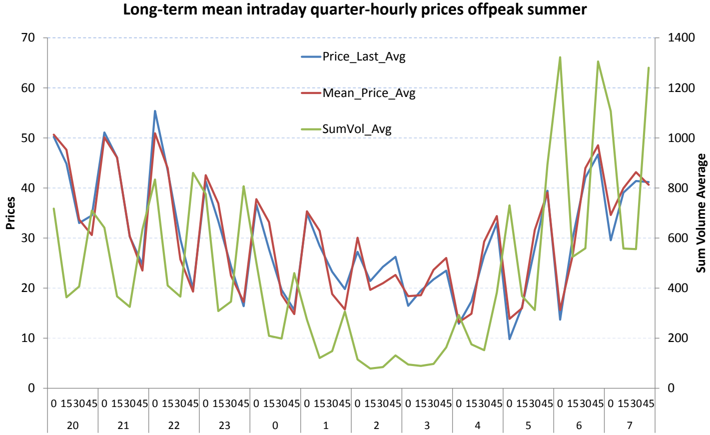
Intra-day Data

- We use quarter-hourly data for the time period 01/01/2014–01/07/2014.
- We aim to model the difference between the last bid price for a certain quarter of an hour and the DayAhead price for that specific hour.
- We observe a jigsaw pattern of the 15-minute Intra-day prices and thus need to control for seasonality.

Seasonality pattern of the last prices and average prices with sunshine duration



Seasonality pattern of the last prices and average prices with volume traded – summer



Fundamental variables

- We employed historical Day-ahead and Intraday electricity prices for 15-minute products in the continuous trading system between 01/01/2014–01/07/2014.
- As fundamental variables we use
  - demand forecast,
  - power plant availability (PPA),
  - intraday updated forecasts for wind,
  - intraday updated forecasts for photovoltaic,
  - volume of trades in the continuous trading,
  - the control area balance.

Data granularity of fundamental variables

Variable	Daily	Hourly	quarter-hourly
DayAhead Price		×	
Intra-day Price			×
Intra-day Volume Trades			×
Wind Forecast			×
PV Forecast			×
Expected Power Plant Availability	×		
Expected Demand		×	
Control area balance			×

Technical specification Threshold Model

The technical specification of our model follows Hansen (2000) and reads:

$$y_i = \theta_1^{'} x_i + \varepsilon_i, \quad \omega_i \leq \tau, \tag{1}$$

$$y_i = \theta_2^{'} x_i + \varepsilon_i, \quad \omega_i > \tau, \tag{2}$$

where ω_i is the threshold variable used to split the sample into two regimes.
To determine the location of the most likely threshold, we will apply Hansen's grid search.
The regression parameters are $(\theta, \lambda_n, \tau)$. Let

$$S_n(\theta, \lambda, \tau) = (Y - X\theta - X(\tau)\lambda)'(Y - X\theta - X(\tau)\lambda) \tag{3}$$

be the sum of squared errors function. Then, by definition, the LS estimators $\hat{\theta}, \hat{\lambda}, \hat{\tau}$ jointly minimize (3).

Threshold Location: Likelihood Ratio Test

To test the hypothesis $H_0 : \tau = \tau_0$, a standard approach is to use the likelihood ratio statistic under the auxiliary assumption that ε_i is i.i.d. $N(0, \sigma^2)$.
Let

$$LR_n(\tau) := n\frac{S_n(\tau) - S_n(\hat{\tau})}{S_n(\hat{\tau})}.$$

The likelihood ratio test of H_0 is to reject for large values of $LR_n(\tau_0)$. Using the $LR_n(\tau)$ function, asymptotic p-values for the likelihood ratio test are derived:

$$p_n = 1 - \left(1 - \exp(-1/2 \cdot LR_n(\tau_0)^2)\right)^2.$$

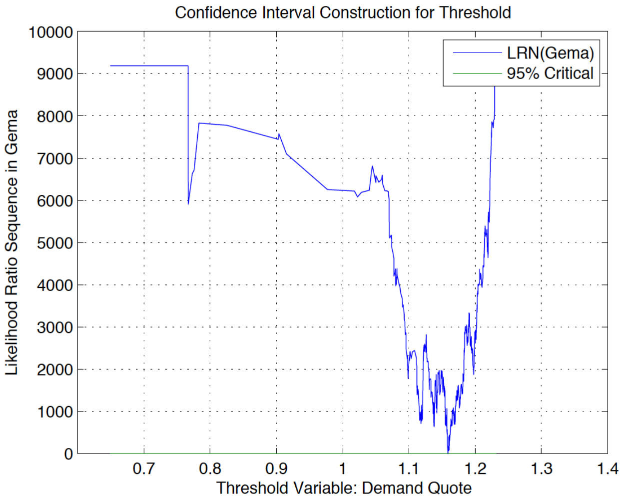
Demand quote

The demand quote is defined as:

$$DemandQuote_t = DemandForecast_t / PPA_{d,t} \tag{4}$$

On average, the demand quote is higher and more volatile during peak than in off-peak hours, which makes the planning of traditional capacity for the day-ahead more difficult.

Threshold Location: Likelihood Ratio Test



Modeling deviations of last prices from the DayAhead price

- We analyze the differences between the historical last prices bid for a certain 15-minute delivery period in the Intra-day market and the DayAhead price for the corresponding hour.
- We include as explanatory variables positive/negative forecasting errors in wind and PV, defined as deviations between the latest forecast available at the time when the last prices are observed and the day-ahead available forecasts.
- We include also a forecast of the control area balance for the given 15-minute delivery period.

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Model specification

- $$(P_t^{ID} - P_t^{Dahd})^h = c^h + \beta^h ControlAreaBalance_t \mathbf{1}_t^h + \theta^h DemandQuote_t \mathbf{1}_t^h + k^{hn}(Wind_t^{ID} - Wind_t^{Dahd})\mathbf{1}_t^h \mathbf{1}_t^n + k^{hp}(Wind_t^{ID} - - Wind_t^{Dahd})\mathbf{1}_t^h \mathbf{1}_t^p + k^{hn}(PV_t^{ID} - PV_t^{Dahd})\mathbf{1}_t^h \mathbf{1}_t^n + k^{hp}(PV_t^{ID} - PV_t^{Dahd})\mathbf{1}_t^h \mathbf{1}_t^p + \sum_{j=1}^{8} \delta_j^h DQ_j \quad (5)$$
- As threshold variable, the DemandQuote splits the data in two regimes: highDemandQuote ("h") or low ("l"), which gives a second equation of the same type.
- The indicator function $\mathbf{1}_t^{p/n}$ further distinguishes in each regime among positive/negative forecasting errors in the renewables.

Model for the continuous trades for quarter-hourly products

- We investigate how delta bid prices change when new information on wind and PV for a certain delivery period of interest becomes available intra-day.
- We look at the trade-off between autoregressive terms and fundamental factors impacting the intra-day price formation process.

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Model specification

$$\begin{aligned}(\Delta P_t^{ID})^h = {} & c^h + \alpha_1^h \Delta P_{t-1}^{ID} \mathbf{1}_t^h + \alpha_2^h \Delta P_{t-2}^{ID} \mathbf{1}_t^h + \alpha_3^h \Delta P_{t-3}^{ID} \mathbf{1}_t^h \\ & + k_w^{hn} (\Delta Wind_t^{ID}) \mathbf{1}_t^h \mathbf{1}_t^n + k_w^{hp} (\Delta Wind_t^{ID}) \mathbf{1}_t^h \mathbf{1}_t^p \\ & + k_{PV}^{hn} (\Delta PV_t^{ID}) \mathbf{1}_t^h \mathbf{1}_t^n + k_{PV}^{hp} (\Delta PV_t^{ID}) \mathbf{1}_t^h \mathbf{1}_t^p \\ & + \gamma^h DemandQuote_t^{Dahd} \mathbf{1}_t^h + \epsilon^h Volume_t^{ID} \mathbf{1}_t^h + \beta_h \sqrt{\Delta t} \\ (\Delta P_t^{ID})^l = {} & c^l + \alpha_1^l \Delta P_{t-1}^{ID} \mathbf{1}_t^l + \alpha_2^l \Delta P_{t-2}^{ID} \mathbf{1}_t^l + \alpha_3^l \Delta P_{t-3}^{ID} \mathbf{1}_t^l \\ & + k_w^{ln} (\Delta Wind_t^{ID}) \mathbf{1}_t^l \mathbf{1}_t^n + k_w^{lp} (\Delta Wind_t^{ID}) \mathbf{1}_t^l \mathbf{1}_t^p \\ & + k_{PV}^{ln} (\Delta PV_t^{ID}) \mathbf{1}_t^l \mathbf{1}_t^n + k_{PV}^{lp} (\Delta PV_t^{ID}) \mathbf{1}_t^l \mathbf{1}_t^p \\ & + \gamma^l DemandQuote_t^{Dahd} \mathbf{1}_t^l + \epsilon^l Volume_t^{ID} \mathbf{1}_t^l + \beta_l \sqrt{\Delta t} \end{aligned} \tag{6}$$

Model specification II

- We find that the first 3 lags of price changes should be selected in the autoregressive part of the model.
- Changes in the wind $\Delta Wind_t^{ID}$ and in PV ΔPV_t^{ID} are real time updated forecasts, available at the time when the bids are placed.
- $Volume_t^{ID}$ is the volume of trade at the time when the price change is observed.
- As the bids for a certain quarter of an hour do not occur at equal time intervals, we include the control variable $\sqrt{\Delta t}$ in our list of explanatory variables.

Agenda

Intraday Trading

Model Architecture

Results

- DayAhead vs Intra-day Last
- Price Delta Continuous Trading

Estimation Results – Global OLS

- The model has been estimated for the historical differences between the last prices and the day-ahead prices separately for winter and summer,
- and peak (8 am to 8 pm) and off-peak hours.
- We tested for a threshold effect in the demand quote in each case using Hansen's methodology.
- We found evidence for significant threshold effect only in the case of the winter peak case study.

Estimation results, Model 1 global OLS without threshold

Dependent variable Delta Last Price- Price DayAhead

	Summer off-peak		Summer peak		Winter off-peak		Winter peak	
	Coeff	Std. err.	Coeff	Std. err.	Coeff	Std. err.	Coeff	Std. err.
Co	7.388	1.971	-20.956	6.128	14.469	4.762	-9.015	10.354
DemandQ	-7.438	2.159	10.929	4.852	-12.715	4.605	-0.354	8.728
Balancing	0.007	0.001	0.008	0.001	0.014	0.001	0.009	0.001
DeltaWindP	-0.005	0.001	-0.002	0.001	-0.003	0.001	-0.003	0.001
DeltaWindN	-0.007	0.001	-0.012	0.001	-0.004	0.001	-0.004	0.001
DeltaPVP	–	–	-0.003	0.001	–	–	-0.003	0.001
DeltaPVN	–	–	-0.004	0.001	–	–	-0.005	0.001
DQ1M	10.170	1.112	10.022	1.462	-4.561	1.729	23.808	2.340
DQ2M	3.515	1.144	2.192	1.507	-5.094	1.717	11.336	2.148
DQ3M	-6.519	1.122	-1.486	1.463	-3.148	1.704	2.740	2.207
DQ4M	-10.454	1.139	-6.031	1.622	-1.187	1.719	-0.548	2.296
DQ1A	-13.845	1.219	-8.111	1.539	3.114	1.848	-6.098	2.173
DQ2A	-6.852	1.229	0.268	1.374	-0.948	1.802	3.203	2.016
DQ3A	0.349	1.161	3.458	1.341	-4.578	1.793	16.773	2.118
DQ4A	4.842	1.203	13.132	1.451	-4.568	1.825	25.588	2.294
Rsquared	35.43%		37.99%		28.76%		36.63%	
No. Obs.	2543		2483		2447		2363	

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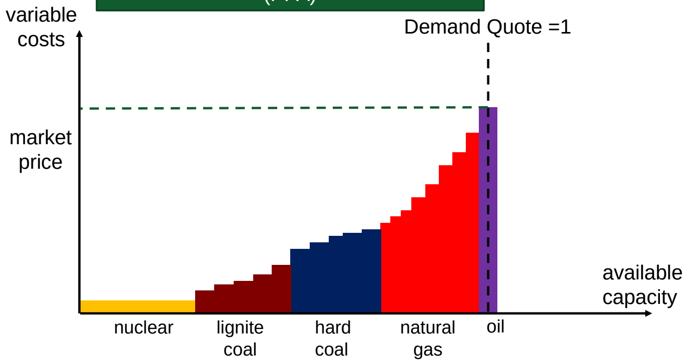
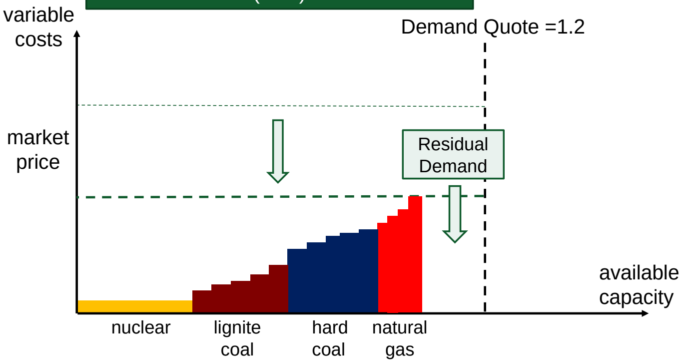
Planned traditional capacity day-ahead (PPA)



The traditional planned capacity for the day-ahead covers fully the expected demand for electricity. There is no (very low) market expectation of renewables.

Planned traditional capacity day-ahead (PPA)



The traditional planned capacity for the day-ahead does not fully cover the expected demand, since market participants expect (up to 20%) renewables infeed in the market. The price is expected to decrease.

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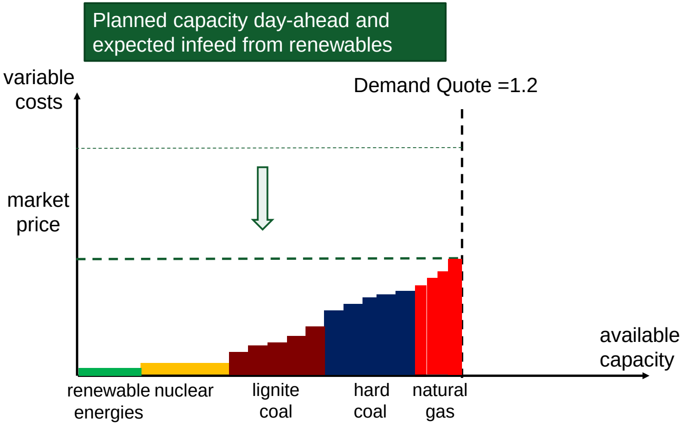
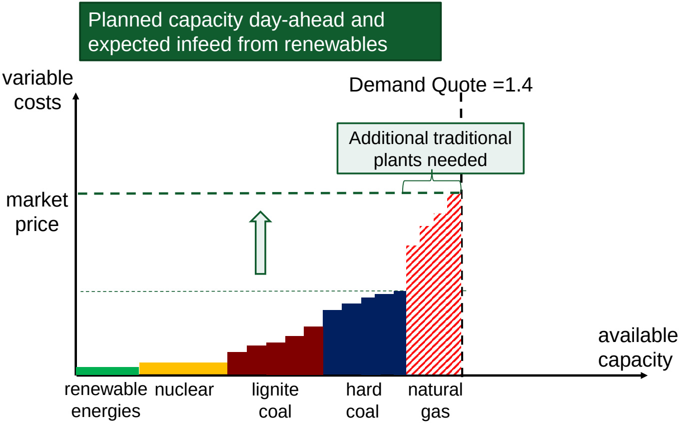


The renewables infeed decreases residual demand and supresses intraday prices.



If the renewables infeed does not supplement the excess demand, bidding for additional traditional capacity intraday can become costly: upwards pressure on the intraday prices.

Winter peak, Threshold variable: Demand Quote

Threshold estimation (threshold variable DemandQ)
Dependent variable Delta Last Price- Price DayAhead

	Regime 1		**Regime 2**	
Threshold value	$<=$ 1.158		$>$ 1.158	
	Coeff	Std. Err.	Coeff	Std. Err.
Co	-48.973	15.527	63.563	22.987
DemandQ	26.810	12.806	-61.545	19.412
Regelzonesaldo	0.003	0.002	0.010	0.001
DeltaWindP	-0.004	0.003	-0.002	0.001
DeltaWindN	-0.006	0.003	-0.004	0.001
DeltaPVP	-0.003	0.002	-0.004	0.001
DeltaPVN	-0.006	0.001	-0.006	0.001
DQ1M	41.322	8.710	21.500	2.324
DQ2M	21.880	7.985	10.443	2.129
DQ3M	4.806	7.948	3.682	2.205
DQ4M	2.266	8.284	0.298	2.329
DQ1A	-8.175	7.420	-1.367	2.340
DQ2A	8.898	7.325	3.440	2.207
DQ3A	30.651	7.536	12.192	2.235
DQ4A	45.249	7.616	17.453	2.369
Rsquared	48.61%		35.93%	
No. Obs.	652		1711	

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Estimation Results – Winter Peak Threshold

- In the regime of low levels of demand quote (regime 1, $DQ < 1.158$) coefficients are generally not statistically significant. Thus, traditional capacity satisfies demand.
- In regime 2 delta prices adjust linearly to forecasting errors in renewable energy, to control area balances and to demand quote.
- The coefficient of control area balances is positive and significant. So, a decrease in the forecasts of control area balances will suppress the Intra-day last prices and decrease the Delta, while positive forecasts in control area balances will increase Intra-day prices and so the Delta.

Autoregressive Terms vs Fundamentals

- Fundamentals become more important during peak hours
- Effect increases with high share of renewables
- Threshold effects only in Demand Quote

Hour 7, global OLS without threshold

OLS estimation of the model including fundamental variables
Dependent variable Delta Price

	H7Q1		H7Q2		H7Q3		H7Q4	
	Coeff	Std. err.	Coeff	Std. err.	Coeff	Std. err.	Coeff	Std. err.
Co	0.288	(0.645)	-0.450	(0.965)	-1.392	(1.139)	-1.102	(0.858)
DeltaPrice1	-0.208*	(0.030)	-0.320*	(0.032)	-0.244*	(0.035)	-0.281*	(0.033)
DeltaPrice2	-0.157*	(0.032)	-0.159*	(0.021)	-0.121*	(0.027)	-0.175*	(0.020)
DeltaPrice3	-0.084*	(0.017)	-0.080*	(0.018)	-0.084*	(0.019)	-0.086*	(0.016)
DemandQuote	-0.300	(0.543)	0.381	(0.829)	0.966	(0.965)	1.011	(0.736)
Volume	0.008	(0.005)	0.015	(0.009)	0.001	(0.009)	-0.020*	(0.006)
SqrTimeStep	-0.833	(1.420)	-1.212	(1.359)	4.101*	(1.319)	4.127*	(1.547)
DeltaWindIntrP	0.0001	(0.0002)	0.0002	(0.0002)	-0.001	(0.001)	-0.001	(0.001)
DeltaWindIntrN	-0.001*	(0.0001)	0.0001	(0.0002)	0.0002	(0.001)	0.001	(0.001)
DeltaPVIntraP	0.0001	(0.001)	0.001	(0.001)	0.0002	(0.001)	0.002	(0.002)
DeltaPVIntraN	0.001	(0.001)	0.002**	(0.001)	-0.001	(0.001)	0.000	(0.001)
Rsquared	5.989%		10.930%		7.333%		9.481%	
No. Obs.	6979		4873		4977		7175	

OLS estimation of the autoregressive model, excluding fundamental variables
Dependent variable Delta Price

	H7Q1		H7Q2		H7Q3		H7Q4	
	Coeff	Std. err.	Coeff	Std. err.	Coeff	Std. err.	Coeff	Std. err.
Co	0.004	(0.061)	0.005	(0.086)	0.010	(0.086)	0.007	(0.072)
DeltaPrice1	-0.207*	(0.012)	-0.321*	(0.014)	-0.243*	(0.014)	-0.276*	(0.012)
DeltaPrice2	-0.158*	(0.012)	-0.159*	(0.015)	-0.119*	(0.014)	-0.175*	(0.012)
DeltaPrice3	-0.083*	(0.012)	-0.080*	(0.014)	-0.085*	(0.014)	-0.082*	(0.012)
Rsquared	5.055%		9.718%		6.170%		8.085%	
No. Obs.	6979		4873		4977		7175	

Hour 12, global OLS without threshold

OLS estimation of the model including fundamental variables
Dependent variable Delta Price

	H12Q1		H12Q2		H12Q3		H12Q4	
	Coeff	Std. err.	Coeff	Std. err.	Coeff	Std. err.	Coeff	Std. err.
Co	-0.558	(0.672)	-0.674	(0.977)	-0.111	(0.765)	-0.032	(0.799)
DeltaPrice1	-0.175**	(0.086)	-0.167*	(0.043)	-0.207*	(0.038)	-0.140*	(0.020)
DeltaPrice2	-0.071**	(0.032)	-0.040	(0.023)	-0.077**	(0.036)	-0.079*	(0.020)
DeltaPrice3	-0.102	(0.060)	-0.018	(0.017)	-0.039	(0.021)	-0.020	(0.013)
DemandQuote	0.109	(0.499)	0.408	(0.755)	0.156	(0.578)	0.088	(0.635)
Volume	0.053*	(0.019)	0.012	(0.009)	-0.012	(0.009)	-0.013**	(0.006)
SqrTimeStep	0.423	(1.570)	1.868	(1.365)	1.010	(1.348)	1.683	(1.853)
DeltaWindIntrP	-0.001*	(0.000)	-0.001	(0.001)	-0.001*	(0.000)	-0.001*	(0.000)
DeltaWindIntrN	-0.001*	(0.000)	-0.001	(0.001)	-0.001	(0.001)	-0.002**	(0.001)
DeltaPVIntraP	-0.002**	(0.001)	-0.002**	(0.001)	-0.002**	(0.001)	-0.004*	(0.001)
DeltaPVIntraN	0.000	(0.001)	-0.001	(0.001)	-0.002**	(0.001)	-0.002**	(0.001)
Rsquared	7.296%		4.705%		7.011%		8.411%	
No. Obs.	6859		5449		6558		7931	

OLS estimation of the autoregressive model excluding fundamental variables
Dependent variable Delta Price

	H12Q1		H12Q2		H12Q3		H12Q4	
	Coeff	Std. err.	Coeff	Std. err.	Coeff	Std. err.	Coeff	Std. err.
Co	0.006	(0.077)	0.004	(0.099)	0.005	(0.092)	0.003	(0.066)
DeltaPrice1	-0.172*	(0.012)	-0.167*	(0.014)	-0.206*	(0.012)	-0.137*	(0.011)
DeltaPrice2	-0.065*	(0.012)	-0.041*	(0.014)	-0.077*	(0.013)	-0.078*	(0.011)
DeltaPrice3	-0.099*	(0.012)	-0.018	(0.014)	-0.041*	(0.012)	-0.019	(0.011)
Rsquared	3.715%		2.733%		4.219%		2.187%	
No. Obs.	6859		5449		6558		7931	

Hour 18, global OLS without threshold

OLS estimation of the model including fundamental variables
Dependent variable Delta Price

	H18Q1		H18Q2		H18Q3		H18Q4	
	Coeff	Std. err.	Coeff	Std. err.	Coeff	Std. err.	Coeff	Std. err.
Co	-0.156	(0.809)	0.068	(0.941)	-1.861	(0.980)	-1.160	(1.087)
DeltaPrice1	-0.206*	(0.032)	-0.276*	(0.036)	-0.254*	(0.033)	-0.214*	(0.036)
DeltaPrice2	-0.163*	(0.033)	-0.149*	(0.025)	-0.173*	(0.030)	-0.105*	(0.023)
DeltaPrice3	-0.131*	(0.024)	-0.090*	(0.024)	-0.101*	(0.020)	-0.149*	(0.045)
DemandQuote	0.324	(0.642)	0.186	(0.772)	1.274	(0.806)	0.708	(0.908)
Volume	-0.025*	(0.004)	-0.028*	(0.006)	0.041*	(0.007)	0.037*	(0.005)
SqrTimeStep	0.143	(1.319)	-1.628	(1.062)	-0.233	(0.921)	-3.565*	(1.258)
DeltaWindIntrP	0.000	(0.000)	0.000	(0.000)	-0.001*	(0.000)	0.000	(0.000)
DeltaWindIntrN	-0.003*	(0.001)	-0.001	(0.001)	-0.001	(0.001)	-0.001	(0.001)
DeltaPVIntraP	0.011	(0.009)	-0.006	(0.013)	-0.004	(0.011)	-0.055	(0.033)
DeltaPVIntraN	-0.014**	(0.007)	0.004	(0.011)	-0.012	(0.027)	0.087	(0.105)
Rsquared	11.135%		8.929%		8.048%		7.037%	
No. Obs.	8507		5982		6162		8936	

OLS estimation of the autoregressive model excluding fundamental variables
Dependent variable Delta Price

	H18Q1		H18Q2		H18Q3		H18Q4	
	Coeff	Std. err.	Coeff	Std. err.	Coeff	Std. err.	Coeff	Std. err.
Co	-0.005	(0.058)	-0.001	(0.073)	0.005	(0.082)	0.005	(0.078)
DeltaPrice1	-0.201*	(0.011)	-0.276*	(0.013)	-0.252*	(0.013)	-0.207*	(0.010)
DeltaPrice2	-0.163*	(0.011)	-0.146*	(0.013)	-0.170*	(0.013)	-0.100*	(0.011)
DeltaPrice3	-0.131*	(0.011)	-0.088*	(0.013)	-0.098*	(0.013)	-0.144*	(0.010)
Rsquared	6.099%		7.715%		7.247%		5.859%	
No. Obs.	8507		5982		6162		8936	

Threshold Effects

- Demand Quote can be identified as threshold
- Hour 12: Wind/ PV forecasting errors only significant in high demand quote regime
- Hour 7 and 18: in high demand quote regime volume of trades becomes significant.

Hour 12, Quarters 1–4, First Sample Split

Regime 1

	H12Q1		**H12Q2**		**H12Q3**		**H12Q4**	
Threshold value	<= 1.245∗		<= 1.245∗		<= 1.146∗		<= 1.197∗	
	Coeff	Std. err.	Coeff	Std. err.	Coeff	Std. err.	Coeff	Std. err.
Co	-0.669	(1.982)	-0.693	(3.302)	0.421	(2.418)	0.365	(3.418)
DeltaPrice1	-0.202	(0.118)	-0.126*	(0.043)	-0.191**	(0.075)	-0.108*	(0.031)
DeltaPrice2	-0.065	(0.043)	-0.042**	(0.021)	-0.142	(0.085)	-0.082**	(0.040)
DeltaPrice3	-0.099	(0.078)	-0.010	(0.018)	-0.023	(0.078)	-0.030	(0.017)
DemandQuote	0.163	(1.685)	0.518	(2.798)	0.036	(2.104)	-0.378	(3.069)
Volume	0.070**	(0.028)	0.022	(0.012)	-0.007	(0.029)	0.003	(0.016)
SqrTimeStep	-1.363	(2.119)	-0.205	(1.886)	-9.905	(5.560)	0.880	(2.436)
DeltaWindIntrP	0.000	(0.001)	0.000	(0.001)	0.005*	(0.002)	-0.001	(0.001)
DeltaWindIntrN	-0.001	(0.001)	-0.001	(0.001)	-0.006*	(0.001)	0.002	(0.002)
DeltaPVIntraP	-0.003*	(0.001)	-0.003*	(0.001)	-0.007**	(0.003)	-0.002	(0.002)
DeltaPVIntraN	0.001	(0.001)	-0.001	(0.001)	-0.002	(0.002)	-0.003*	(0.001)
Rsquared	9.155%		3.806%		27.371%		7.764%	
No. Obs.	3911		3052		487		2438	

Regime 2

	H12Q1		**H12Q2**		**H12Q3**		**H12Q4**	
Threshold value	> 1.245∗		> 0.757∗		> 1.146∗		> 1.197∗	
	Coeff	Std. err.	Coeff	Std. err.	Coeff	Std. err.	Coeff	Std. err.
Co	0.125	(1.349)	-1.036	(1.809)	-0.037	(0.928)	0.405	(0.944)
DeltaPrice1	-0.094**	(0.040)	-0.256*	(0.060)	-0.208*	(0.040)	-0.155*	(0.022)
DeltaPrice2	-0.108	(0.040)	-0.046	(0.053)	-0.072	(0.038)	-0.075	(0.020)
DeltaPrice3	-0.099**	(0.043)	-0.035	(0.035)	-0.039	(0.022)	-0.011	(0.018)
DemandQuote	-0.216	(0.965)	0.630	(1.304)	0.065	(0.693)	-0.163	(0.692)
Volume	0.018**	(0.008)	-0.006	(0.013)	-0.012	(0.010)	-0.021*	(0.006)
SqrTimeStep	1.140	(1.439)	3.942**	(1.758)	2.263	(1.191)	-0.097	(1.700)
DeltaWindIntrP	-0.002*	(0.000)	-0.002**	(0.001)	-0.001*	(0.000)	-0.001	(0.001)
DeltaWindIntrN	-0.001*	(0.000)	-0.002**	(0.001)	-0.001	(0.001)	-0.002**	(0.001)
DeltaPVIntraP	0.000	(0.001)	-0.001	(0.001)	-0.002**	(0.001)	-0.002**	(0.001)
DeltaPVIntraN	-0.001	(0.001)	-0.002**	(0.001)	-0.001	(0.001)	-0.004*	(0.001)
Rsquared	8.868%		10.760%		6.590%		11.624%	
No. Obs.	2948		2397		6071		5493	

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Hour 7, Quarters 1–4, First Sample Split, Regime 1

Dependent variable Delta Price								
	H7Q1		H7Q2		H7Q3		H7Q4	
Regime 1								
Threshold value	$<= 1.161*$		$<= 0.757*$		$<= 0.828*$		$<= 1.415*$	
	Coeff	Std. err.	Coeff	Std. err.	Coeff	Std. err.	Coeff	Std. err.
Co	0.765	(1.365)	16.416*	(7.688)	-16.689	(13.279)	-1.561**	(0.822)
DeltaPrice1	-0.184*	(0.036)	-0.155**	(0.073)	-0.221*	(0.083)	-0.255*	(0.030)
DeltaPrice2	-0.193*	(0.038)	-0.187*	(0.044)	-0.087	(0.085)	-0.169*	(0.020)
DeltaPrice3	-0.098*	(0.022)	-0.005	(0.051)	-0.075	(0.057)	-0.086*	(0.017)
DemandQuote	-0.844	(1.253)	-21.980**	(10.706)	19.229	(17.252)	1.416**	(0.700)
Volume	0.010	(0.007)	0.044	(0.108)	-0.061	(0.053)	-0.018*	(0.006)
SqrTimeStep	0.054	(1.959)	1.370	(9.574)	44.873*	(12.333)	3.820**	(1.571)
DeltaWindIntrP	0.000	(0.000)	-0.056*	(0.018)	-0.134*	(0.025)	-0.001	(0.001)
DeltaWindIntrN	0.000	(0.001)	-0.013	(0.017)	0.014**	(0.007)	0.001	(0.001)
DeltaPVIntraP	0.001	(0.002)	0.001	(0.013)	0.007	(0.024)	0.003*	(0.001)
DeltaPVIntraN	0.000	(0.001)	0.012	(0.011)	0.011	(0.008)	0.000	(0.001)
Rsquared	6.081%		67.460%		63.497%		9.053%	
No. Obs.	4090		82		111		6984	

Hour 7, Quarters 1–4, First Sample Split, Regime 2

	H7Q1		H7Q2		H7Q3		H7Q4	
Regime 2								
Threshold value	$> 1.161*$		$> 0.757*$		$> 0.828*$		$> 1.415*$	
	Coeff	Std. err.	Coeff	Std. err.	Coeff	Std. err.	Coeff	Std. err.
Co	0.388	(1.305)	-0.368	(1.062)	-0.172	(1.095)	-58.038	(120.183)
DeltaPrice1	-0.233*	(0.050)	-0.318*	(0.031)	-0.236*	(0.035)	-0.363*	(0.135)
DeltaPrice2	-0.081	(0.049)	-0.156*	(0.022)	-0.109*	(0.020)	-0.231*	(0.088)
DeltaPrice3	-0.047	(0.025)	-0.084*	(0.019)	-0.081*	(0.018)	-0.093**	(0.047)
DemandQuote	-0.210	(1.023)	0.302	(0.904)	-0.096	(0.914)	39.713	(83.769)
Volume	0.004	(0.006)	0.014	(0.009)	0.002	(0.009)	-0.035	(0.039)
SqrTimeStep	-3.034	(1.930)	-0.905	(1.372)	4.528*	(1.291)	43.401*	(17.220)
DeltaWindIntrP	-0.002**	(0.001)	0.000	(0.000)	-0.001	(0.001)	-0.052	(0.036)
DeltaWindIntrN	-0.001	(0.001)	0.000	(0.000)	0.000	(0.001)	-0.006	(0.036)
DeltaPVIntraP	0.001	(0.002)	0.001	(0.001)	0.000	(0.001)	-0.029*	(0.004)
DeltaPVIntraN	0.001	(0.001)	0.002**	(0.001)	-0.001	(0.001)	-0.027	(0.055)
Rsquared	10.094%		10.659%		7.349%		47.604%	
No. Obs.	2889		4791		4850		191	

Hour 18, Quarters 1–4, First Sample Split

Regime 1

	H18Q1		H18Q2		H18Q3		H18Q4	
Threshold value	$<= 0.915*$		$<= 1.221*$		$<= 1.219*$		$<= 1.442*$	
	Coeff	Std. err.	Coeff	Std. err.	Coeff	Std. err.	Coeff	Std. err.
Co	46.694	(152.240)	0.020	(2.024)	-5.932*	(2.012)	-0.481	(1.031)
DeltaPrice1	-0.510*	(0.116)	-0.258*	(0.035)	-0.252*	(0.032)	-0.198*	(0.037)
DeltaPrice2	-0.284*	(0.105)	-0.197*	(0.030)	-0.154*	(0.028)	-0.088*	(0.022)
DeltaPrice3	-0.137	(0.086)	-0.079**	(0.031)	-0.111*	(0.029)	-0.148*	(0.049)
DemandQuote	-52.391	(170.802)	0.296	(1.758)	4.995*	(1.757)	0.142	(0.855)
Volume	-0.051	(0.085)	-0.038*	(0.008)	0.041*	(0.008)	0.035*	(0.005)
SqrTimeStep	6.124	(19.295)	-1.137	(1.179)	-0.772	(1.032)	-3.303*	(1.266)
DeltaWindIntrP	0.019	(0.026)	0.000	(0.000)	-0.001*	(0.000)	0.000	(0.000)
DeltaWindIntrN	-0.027	(0.020)	-0.001	(0.001)	0.000	(0.000)	-0.001	(0.001)
DeltaPVIntraP	-0.340	(0.224)	0.038	(0.052)	-0.006	(0.014)	-0.053	(0.032)
DeltaPVIntraN	0.159	(0.321)	0.024	(0.029)	-0.036	(0.045)	0.086	(0.106)
Rsquared	30.618%		8.668%		8.109%		6.356%	
No. Obs.	133		3571		3553		8776	

Regime 2

	H18Q1		H18Q2		H18Q3		H18Q4	
Threshold value	$> 0.915*$		$> 1.221*$		$> 1.219*$		$> 1.442*$	
	Coeff	Std. err.	Coeff	Std. err.	Coeff	Std. err.	Coeff	Std. err.
Co	0.460	(0.670)	0.944	(2.590)	-1.882	(3.752)	-10.224	(43.509)
DeltaPrice1	-0.181*	(0.025)	-0.284*	(0.064)	-0.247*	(0.061)	0.008	(1.892)
DeltaPrice2	-0.161*	(0.035)	-0.095*	(0.039)	-0.171*	(0.055)	-0.090	(0.990)
DeltaPrice3	-0.119*	(0.023)	-0.098*	(0.035)	-0.106*	(0.029)	-0.011	(0.992)
DemandQuote	-0.165	(0.526)	-0.568	(1.970)	1.163	(2.876)	-39.818	(57.807)
Volume	-0.025*	(0.004)	-0.008	(0.012)	0.042*	(0.014)	0.156	(0.506)
SqrTimeStep	-0.212	(1.319)	-3.076	(1.815)	0.507	(1.533)	-48.774	(122.258)
DeltaWindIntrP	0.000	(0.000)	-0.001	(0.001)	0.000	(0.001)	0.000	(0.043)
DeltaWindIntrN	-0.003*	(0.001)	-0.002**	(0.001)	-0.002*	(0.000)	0.204	(0.301)
DeltaPVIntraP	0.012	(0.009)	-0.010	(0.015)	-0.019	(0.014)	0.332	(7.980)
DeltaPVIntraN	-0.014**	(0.007)	-0.008	(0.013)	0.005	(0.031)	-2.765	(8.155)
Rsquared	11.003%		11.252%		9.295%		25.624%	
No. Obs.	8299		2411		2397		160	

Hour 7, Quarters 2,4, Second Sample Split: Further Thresholds

Regime 1

	H7Q2		**H7Q4**	
Threshold value	$<= 1.145*$		$<= 1.178*$	
	Coeff	Std. err.	Coeff	Std. err.
Co	-4.367	(3.103)	-2.411	(1.412)
DeltaPrice1	-0.431*	(0.041)	-0.287*	(0.036)
DeltaPrice2	-0.242*	(0.032)	-0.180*	(0.023)
DeltaPrice3	-0.132*	(0.029)	-0.082*	(0.019)
DemandQuote	4.051	(2.847)	2.224	(1.284)
Volume	-0.014	(0.013)	-0.019*	(0.007)
SqrTimeStep	1.178	(1.807)	4.501*	(1.694)
DeltaWindIntrP	-0.001*	(0.000)	-0.001	(0.001)
DeltaWindIntrN	-0.001	(0.001)	0.000	(0.001)
DeltaPVIntraP	0.000	(0.002)	-0.001	(0.001)
DeltaPVIntraN	0.001	(0.001)	0.002	(0.002)
Rsquared	18.171%		9.526%	
No. Obs.	2175		4605	

Regime 2

	H7Q2		**H7Q4**	
Threshold value	$> 1.145*$		$> 1.178*$	
	Coeff	Std. err.	Coeff	Std. err.
Co	-0.826	(2.130)	3.845	(4.117)
DeltaPrice1	-0.227*	(0.045)	-0.168*	(0.042)
DeltaPrice2	-0.102*	(0.029)	-0.091*	(0.028)
DeltaPrice3	-0.058**	(0.023)	-0.090	(0.047)
DemandQuote	0.536	(1.724)	-3.034	(3.344)
Volume	0.043*	(0.012)	-0.013	(0.011)
SqrTimeStep	-2.138	(1.950)	1.134	(3.719)
DeltaWindIntrP	0.000	(0.001)	-0.002**	(0.001)
DeltaWindIntrN	0.000	(0.001)	0.001	(0.003)
DeltaPVIntraP	0.001	(0.001)	0.006	(0.004)
DeltaPVIntraN	0.003*	(0.001)	-0.005*	(0.001)
Rsquared	7.361%		19.427%	
No. Obs.	2576		1665	

Extensions + Literature

- E. Garnier, R.Madlener, 2014. *Balancing forecast errors in continuous-trade Intra-day markets.* FCN WP 2/2014, RWTH Aachen University School of Business and Economics.
- *An optimal trading problem in Intra-day electricity markets*, R. Aid, P. Gruet, H.Pham.

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Modelling Day-Ahead and Intraday Electricity Markets

Ökonomik der Energiewende , KIT

Rüdiger Kiesel | Chair for Energy Trading and Finance | University of Duisburg-Essen

Part III

Market Making in the German Intraday Power Market

Motivation

Model

Possible Backtest

Agenda

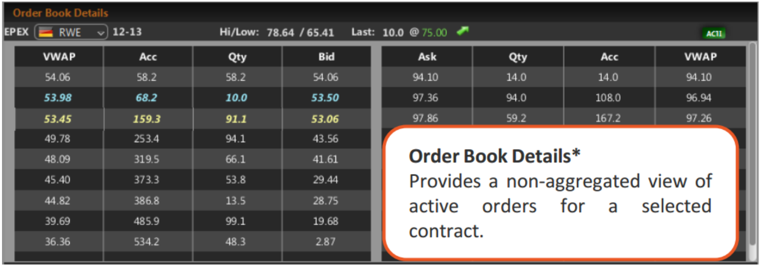
Motivation

Model

Possible Backtest

Limit Order Book

- Sell and buy limit orders (LO)
- Limit order book (LOB) visible to all market participants
- Bid-ask spread is difference between best sell and buy LO, here 40.04 EUR per MW (extremely high)



VWAP	Acc	Qty	Bid	Ask	Qty	Acc	VWAP
54.06	58.2	58.2	54.06	94.10	14.0	14.0	94.10
53.98	68.2	10.0	53.50	97.36	94.0	108.0	96.94
53.45	159.3	91.1	53.06	97.86	59.2	167.2	97.26
49.78	253.4	94.1	43.56				
48.09	319.5	66.1	41.61				
45.40	373.3	53.8	29.44				
44.82	386.8	13.5	28.75				
39.69	485.9	99.1	19.68				
36.36	534.2	48.3	2.87				

Source: `https://www.epexspot.com/document/30313/ComTrader%20-%20Guideline`

Figure: Snapshot of limit order book for product H13.

Orders

- A buy (sell) limit order (LO) is an instrument which allows an agent to express how much she wants to pay (receive) per share for a specific number of shares.
- All unfilled buy and sell limit orders are gathered in the limit-order book (LOB).
- A buy (sell) market order (MO) is an instrument which allows an agent to buy (sell) a specific number of shares at the current best sell (buy) limit order price(s).

Evolution of Best Sell and Buy LO Prices

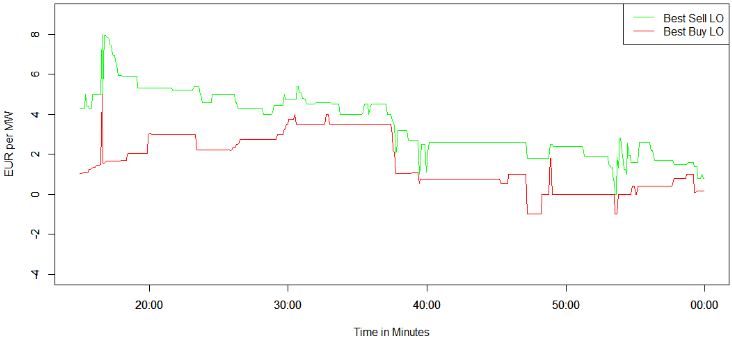


Figure: Best sell and buy limit order prices for product H14 with delivery on April 15, 2015 in the trading window “Late”.

Business of a Market Maker

- Market makers place both sell and buy limit orders, thus allowing other traders to make the trades they desire
- Their intention in placing limit orders on the sell and the buy side of the market is that both are lifted shortly after each other, thus allowing them to realize the bid-ask spread
- If only a sell or a buy limit order placed by a market maker is lifted, she builds up an inventory position; the risk of loosing money due to the impact of a price change on the inventory position is referred to as inventory risk
- If market makers obtain a piece of information which they believe is not contained in the price yet, they may temporarily suspend their limit order operations and instead trade on that piece of information

Agenda

Motivation

Model

Possible Backtest

Market Making Formalized

- Model is inspired by the one presented in [Ric14; CJR14]
- Terminal wealth from market maker operations is denoted X_T; wealth dynamics:

$$dX_t = \left(S_t + \delta_t^+\right) dN_t^+ - \left(S_t - \delta_t^-\right) dN_t^-$$

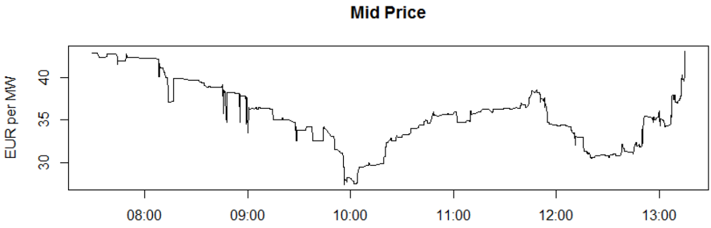
- The makeup (markdown) which the agent adds (subtracts from) the mid price to price her sell (buy) limit orders is labeled δ_t^+ (δ_t^-).
- If a sell (buy) limit order placed by the agent is lifted her inventory decreases (increases).
- The processes N_t^+ and N_t^- represent the number of sell and buy limit orders placed by the agent which have been lifted by buy and sell market orders, respectively.

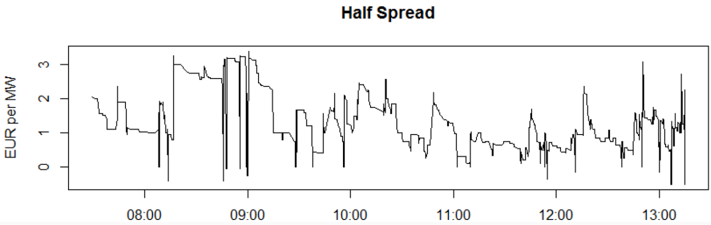
Market Making Formalized II

In addition to X_T market maker's value function comprises liquidation of terminal inventory q_T at mid price S_T plus/minus halfspread H_T plus/minus terminal penalty αq_T and penalty on inventory held during trading period:

$$\Phi = \sup_{\delta_u^+,\delta_u^-} \mathbb{E}\left[X_T + q_T \left(S_T - \operatorname{sgn}(q_T) H_T - \alpha q_T \right) - \phi \int_t^T q_u^2 \, du \mid \mathcal{F}_t \right]$$

Mid Price S and Halfspread H





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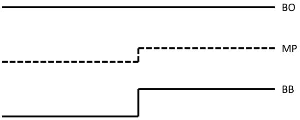
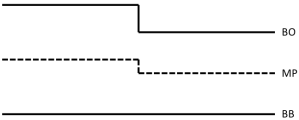
Mid Price *S* and Halfspread *H*

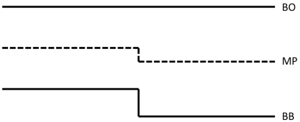
- **Insert Buy LO**:
  S increases by ϵ_{ν}^{+},
  H decreases by ϵ_{ν}^{+}



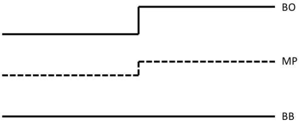
- **Insert Sell LO**:
  S decreases by ϵ_{υ}^{-},
  H decreases by ϵ_{υ}^{-}



- **Cancel Buy LO**:
  S decreases by ϵ_{ν}^{-},
  H increases by ϵ_{ν}^{-}



- **Cancel Sell LO**:
  S increases by ϵ_{υ}^{+},
  H increases by ϵ_{υ}^{+}



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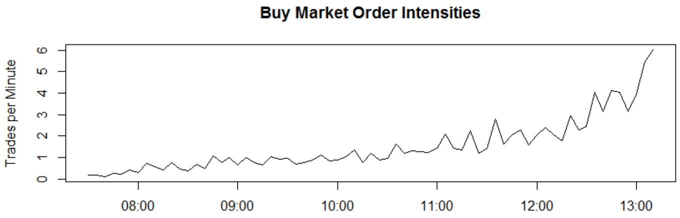
Mid Price Process

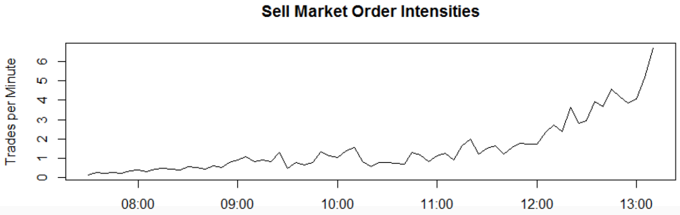
- Mid price dynamics:

$$dS_t = \epsilon_\nu^+ dK_t^+ - \epsilon_\nu^- dK_t^- + \epsilon_\upsilon^+ dL_t^+ - \epsilon_\upsilon^- dL_t^- + \epsilon^+ dM_t^+ - \epsilon^- dM_t^-$$

- $K_t^\pm$ are processes which reflect up and downward changes in MP due to buy LO entering the order book or being canceled
- $L_t^\pm$ are processes which reflect up and downward changes in MP due to sell LO being canceled or entering the order book
- $M_t^\pm$ are processes which reflect up and downward changes in the mid price due to buy or sell market orders
- For $K_t^\pm$, $L_t^\pm$ and $M_t^\pm$ Hawkes processes with conditional intensities $\nu_t^\pm$, $\upsilon_t^\pm$ and $\lambda_t^\pm$ are assumed
- Expectation of $\epsilon_\nu^\pm$, $\epsilon_\upsilon^\pm$ and $\epsilon^\pm$ is $\varepsilon_\nu^\pm$, $\varepsilon_\upsilon^\pm$ and $\varepsilon^\pm$

MO Intensities



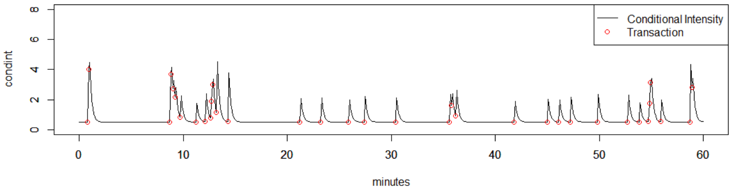


Conditional Intensity $\lambda_t^{\pm}$

- Consider the timestamps of buy and sell market orders $t_i^{\pm}$; then:

$$\lambda_t^+ = \mu^+ + \sum_{t_i^+ < t} \zeta^+ e^{-\rho^+ (t - t_i^+)}$$

$$\lambda_t^- = \mu^- + \sum_{t_i^- < t} \zeta^- e^{-\rho^- (t - t_i^-)}$$



Market Making Formalized

- Hamilton-Jacobi-Bellman (HJB) equation evolves from value function:

$$
\begin{aligned}
0 &= \left(\partial_t + \mathcal{L}\right)\Phi \\
&+ \nu^+ \left[\mathbb{S}^+_\nu \Phi\left(t, X, S + \epsilon^+_\nu, H + \epsilon^+_\nu\right) - \Phi\right] + \nu^- \left[\mathbb{S}^-_\nu \Phi\left(t, X, S - \epsilon^-_\nu, H - \epsilon^-_\nu\right) - \Phi\right] \\
&+ \upsilon^+ \left[\mathbb{S}^+_\upsilon \Phi\left(t, X, S + \epsilon^+_\upsilon, H - \epsilon^+_\upsilon\right) - \Phi\right] + \upsilon^- \left[\mathbb{S}^-_\upsilon \Phi\left(t, X, S - \epsilon^-_\upsilon, H + \epsilon^-_\upsilon\right) - \Phi\right] \\
&\qquad + \lambda^+ \sup_{\delta^+} \Big\{ f\left(\delta^+\right) \left[\mathbb{S}^+_{q\lambda} \Phi\left(t, X + S + \delta^+, S + \epsilon^+, H + \epsilon^+\right) - \Phi\right] \\
&\qquad\qquad + \left(1 - f\left(\delta^+\right)\right) \left[\mathbb{S}^+_\lambda \Phi\left(t, X, S + \epsilon^+, H + \epsilon^+\right) - \Phi\right] \Big\} \\
&\qquad + \lambda^- \sup_{\delta^-} \Big\{ f\left(\delta^-\right) \left[\mathbb{S}^-_{q\lambda} \Phi\left(t, X - S + \delta^-, S - \epsilon^-, H + \epsilon^-\right) - \Phi\right] \\
&\qquad\qquad + \left(1 - f\left(\delta^-\right)\right) \left[\mathbb{S}^-_\lambda \Phi\left(t, X, S - \epsilon^-, H + \epsilon^-\right) - \Phi\right] \Big\} \\
&\qquad\qquad\qquad - \phi q^2
\end{aligned}
$$

- HJB may be used to find optimal markups/downs δ_t^{+*} and δ_t^{-*}

Optimal Markups/downs (1/2)

- Solution:

$$\delta^{+*} = \delta_0^+ + \varepsilon_\nu^+ \delta_{\varepsilon_\nu^+}^+ + \varepsilon_\nu^- \delta_{\varepsilon_\nu^-}^+ + \varepsilon_\upsilon^+ \delta_{\varepsilon_\upsilon^+}^+ + \varepsilon_\upsilon^- \delta_{\varepsilon_\upsilon^-}^+ + \varepsilon^+ \delta_{\varepsilon^+}^+ + \varepsilon^- \delta_{\varepsilon^-}^+ + H\delta_H^+ + \alpha\delta_\alpha^+ + \phi\delta_\phi^+$$

$$\delta^{-*} = \delta_0^- + \varepsilon_\nu^+ \delta_{\varepsilon_\nu^+}^- + \varepsilon_\nu^- \delta_{\varepsilon_\nu^-}^- + \varepsilon_\upsilon^+ \delta_{\varepsilon_\upsilon^+}^- + \varepsilon_\upsilon^- \delta_{\varepsilon_\upsilon^-}^- + \varepsilon^+ \delta_{\varepsilon^+}^- + \varepsilon^- \delta_{\varepsilon^-}^- + H\delta_H^- + \alpha\delta_\alpha^- + \phi\delta_\phi^-$$

- With:

$$\begin{aligned}
\varepsilon_\nu^+ \delta_{\varepsilon_\nu^+}^+ &= \varepsilon_\nu^+ b_{\varepsilon_\nu^+} \\
\varepsilon_\nu^- \delta_{\varepsilon_\nu^-}^+ &= \varepsilon_\nu^- b_{\varepsilon_\nu^-} \\
\varepsilon_\upsilon^+ \delta_{\varepsilon_\upsilon^+}^+ &= \varepsilon_\upsilon^+ b_{\varepsilon_\upsilon^+} \\
\varepsilon_\upsilon^- \delta_{\varepsilon_\upsilon^-}^+ &= \varepsilon_\upsilon^- b_{\varepsilon_\upsilon^-} \\
\varepsilon^+ \delta_{\varepsilon^+}^+ &= \varepsilon^+ \mathbb{S}_\lambda^+ b_{\varepsilon^+} + \varepsilon^+ \left(1 + q\,\mathrm{sgn}\,(q-1) - \mathrm{sgn}\,(q-1) - q\,\mathrm{sgn}\,(q)\right) \\
\varepsilon^- \delta_{\varepsilon^-}^+ &= \varepsilon^- b_{\varepsilon^-} \\
H\delta_H^+ &= H\left(q\,\mathrm{sgn}\,(q-1) - \mathrm{sgn}\,(q-1) - q\,\mathrm{sgn}\,(q)\right) \\
\alpha\delta_\alpha^+ &= \alpha\mathbb{S}_\lambda^+ b_\alpha + \alpha\,(1-2q) \\
\phi\delta_\phi^+ &= \phi\mathbb{S}_\lambda^+ b_\phi + \phi c_\phi\,(-1+2q)
\end{aligned}$$

Optimal Markups/downs (2/2)

- And with:

$$b_{\varepsilon^+_\nu} = \mathbb{E}\left[\int_t^T \nu^+_u \, du\right] = \int_t^T \mathbb{E}\left[\nu^+_u\right] du =$$

$$\frac{\mu^+_\nu \rho^+_\nu}{\left(\varsigma^+_\nu - \rho^+_\nu\right)^2}\left(e^{\left(\varsigma^+_\nu - \rho^+_\nu\right)(T-t)} - 1\right) - \frac{\mu^+_\nu \rho^+_\nu}{\varsigma^+_\nu - \rho^+_\nu}(T-t) + \frac{\nu^+_t}{\varsigma^+_\nu - \rho^+_\nu}\left(e^{\left(\varsigma^+_\nu - \rho^+_\nu\right)(T-t)} - 1\right)$$

...

$$b_{\varepsilon^+_\upsilon} = \mathbb{E}\left[\int_t^T \upsilon^+_u \, du\right] = \int_t^T \mathbb{E}\left[\upsilon^+_u\right] du =$$

$$\frac{\mu^+_\upsilon \rho^+_\upsilon}{\left(\varsigma^+_\upsilon - \rho^+_\upsilon\right)^2}\left(e^{\left(\varsigma^+_\upsilon - \rho^+_\upsilon\right)(T-t)} - 1\right) - \frac{\mu^+_\upsilon \rho^+_\upsilon}{\varsigma^+_\upsilon - \rho^+_\upsilon}(T-t) + \frac{\upsilon^+_t}{\varsigma^+_\upsilon - \rho^+_\upsilon}\left(e^{\left(\varsigma^+_\upsilon - \rho^+_\upsilon\right)(T-t)} - 1\right)$$

...

$$b_{\varepsilon^+} = \mathbb{E}\left[\int_t^T \lambda^+_u \, du\right] = \int_t^T \mathbb{E}\left[\lambda^+_u\right] du =$$

$$\frac{\mu^+ \rho^+}{\left(\varsigma^+ - \rho^+\right)^2}\left(e^{\left(\varsigma^+ - \rho^+\right)(T-t)} - 1\right) - \frac{\mu^+ \rho^+}{\varsigma^+ - \rho^+}(T-t) + \frac{\lambda^+_t}{\varsigma^+ - \rho^+}\left(e^{\left(\varsigma^+ - \rho^+\right)(T-t)} - 1\right)$$

...

Optimal Markup Detail (1/2)

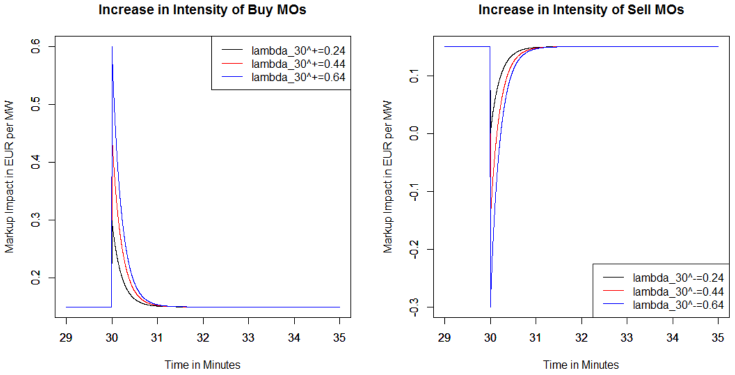


Figure: Impact of increases in the intensity of buy and sell market orders on the sum of the markup components for market order arrivals.

Optimal Markup Detail (2/2)

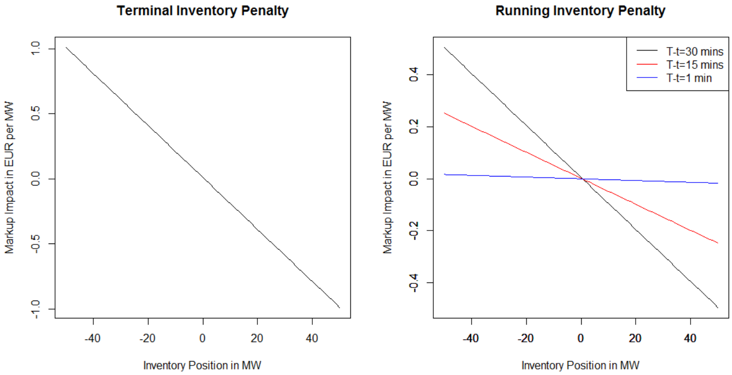


Figure: Impact of changes in the inventory position on the markup component for the terminal inventory penalty and on the markup component for the running inventory penalty.

Agenda

Motivation

Model

Possible Backtest

Backtest Results

Test Description

- Backtest is based on the one presented in Guéant-Lehalle-Tapia (2013).
- Every 5 secs Agent places new 10 MW sell and buy LO
- Impacts of shifts in inventory position and market order intensities are ignored
- Only trading windows last trading hour "Last" and second-last trading hour "Mid" are considered.
- If the pricing logic suggests a price which is lower (higher) than the best sell (buy) LO price, agent places sell (buy) LO at current best sell (buy) LO
- If the agent's sell or buy LO is hit, her inventory is updated and she places new 10 MW sell and buy LO
- LO which has been hit in real life is also removed; other market participants do not react to agent's LOs
- Different scenarios are considered:
  - α: 0, 0.01, 0.05, 0.1 EUR per MW^2
  - ϕ: Such that $\phi\,(T - t)$ is 0, 0.01, 0.1 EUR per MW^2 at the beginning

Example LO Price Paths

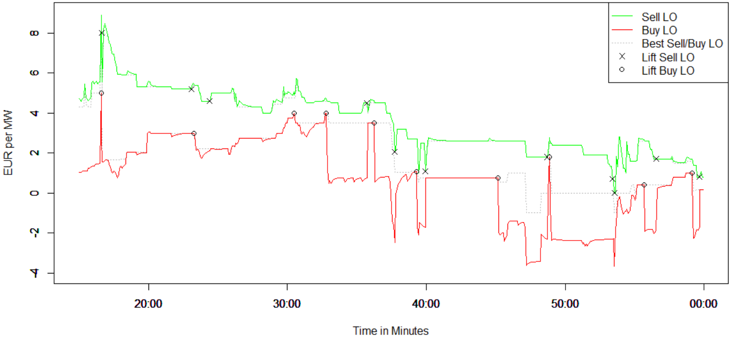


Figure: Optimal sell and buy limit order prices and buy and sell market orders which would have lifted the Agent's sell and buy limit orders as well as best sell and buy limit order prices for product H14 with delivery on April 15, 2015 in the trading window "Late". $\alpha = 0.1$ EUR per MW^2 and $\phi(T-t) = 0.01$ EUR per MW^2 at the beginning of the trading window.

Example Inventory Path

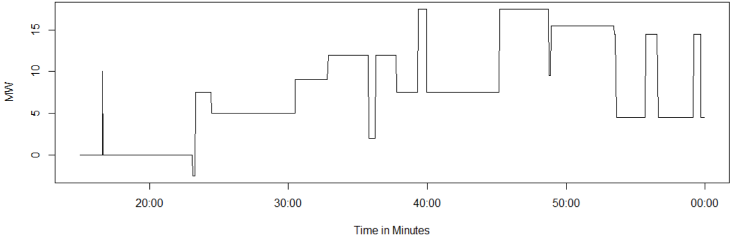


Figure: Inventory position for product H14 with delivery on April 15, 2015 in the trading window "Late". $\alpha = 0.1$ EUR per MW^2 and $\phi(T - t) = 0.01$ EUR per MW^2 at the beginning of the trading window.

Backtest Results for Trading Window “Late”

	Indicator	Measure	Unit	$\alpha = 0$	$\alpha = 0.01$	$\alpha = 0.05$	$\alpha = 0.1$
$\phi\,(45-0)$ $= 0.01$	PnL	mean	EUR	18.63	24.82	27.54	28.71
		std	EUR	98.54	79.68	66.74	65.13
		Q_5	EUR	−143.63	−88.87	−68.48	−60.80
		Q_{95}	EUR	134.44	129.72	123.56	123.93
		skewness	n/a	−1.14	−0.79	0.59	0.72
		kurtosis	n/a	15.52	16.10	12.51	13.12
	Inventory	mean	MW	1.51	1.44	1.21	1.13
		std	MW	25.48	16.79	11.84	10.74
		Q_5	MW	−40.64	−26.04	−16.64	−16.18
		Q_{95}	MW	41.40	28.30	19.24	17.68
		skewness	n/a	−0.14	−0.15	−0.02	−0.06
		kurtosis	n/a	5.19	3.79	2.61	2.14
	Volume	mean	MW	178.17	168.14	158.14	155.23
		std	MW	74.85	70.23	66.49	64.97
		Q_5	MW	74.30	69.12	64.40	63.92
		Q_{95}	MW	321.50	300.94	284.88	277.28
		skewness	n/a	0.78	0.73	0.70	0.69
		kurtosis	n/a	3.69	3.60	3.49	3.53

Table: Backtest results for March 02, 2015 to June 26, 2015 and “Late”

Backtest Results for Trading Window “Mid”

	Indicator	Measure	Unit	$\alpha = 0$	$\alpha = 0.01$	$\alpha = 0.05$	$\alpha = 0.1$
$\phi(60-0) = 0.01$	PnL	mean	EUR	−3.77	1.21	3.84	4.49
		std	EUR	75.74	60.40	46.33	43.72
		Q_5	EUR	−115.36	−87.45	−70.30	−65.56
		Q_{95}	EUR	81.81	77.23	73.59	72.31
		skewness	n/a	−4.30	−2.65	−0.99	−0.64
		kurtosis	n/a	47.14	20.04	8.08	5.73
	Inventory	mean	MW	1.47	0.72	0.69	0.82
		std	MW	22.14	14.96	11.35	10.83
		Q_5	MW	−34.08	−25.00	−17.64	−17.0
		Q_{95}	MW	39.52	23.16	18.00	17.8
		skewness	n/a	0.02	−0.22	−0.16	−0.18
		kurtosis	n/a	4.90	3.22	2.16	2.02
	Volume	mean	MW	109.79	102.83	96.68	95.43
		std	MW	57.19	53.16	50.26	49.91
		Q_5	MW	32.24	30.30	30.00	30.00
		Q_{95}	MW	213.40	200.94	183.72	183.02
		skewness	n/a	1.03	0.99	1.02	1.05
		kurtosis	n/a	4.80	4.63	5.02	5.17

Table: Backtest results for March 02, 2015 to June 26, 2015 and “Mid”

Results/Interpretation

- The means of the PnLs indicate that no matter if the agent prices a terminal and/or a running inventory penalty into her sell and buy limit order prices or not, the market making strategy is expected to allow her earning money.
- The means of the PnLs largely decrease with decreasing terminal and running inventory penalty.
  - The agent's protection against being lifted repeatedly on the same side of the market decreases with decreasing terminal and running inventory penalty. So the downside of being lifted repeatedly on one side of the market market and unwinding the resulting inventory position at unfavorable prices outweighs the upside of being lifted repeatedly on one side of the market market and unwinding the resulting inventory position at favorable prices.
  - The agent's prices are more favorable for her the greater the terminal and the running inventory penalty are.

Relevant Distributions

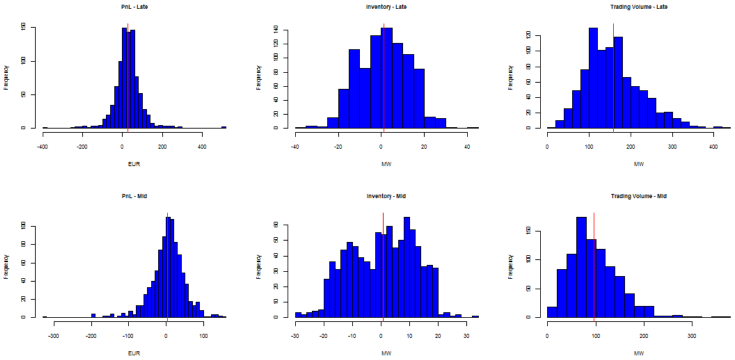


Figure: Distribution of PnLs, terminal inventory positions and trading volumes for the trading windows "Late" and "Mid". $\alpha = 0.05$ EUR per MW^2 and $\phi(T-t) = 0.01$ EUR per MW^2 at the beginning of a trading window.

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Contact

Chair for Energy Trading and Finance
University of Duisburg-Essen
Universitätsstraße 12
GER-45141 Essen

Phone: +49 (0)201 183-4973
Fax: +49 (0)201 183-4974

Web: `www.lef.wiwi.uni-due.de`

Thank you for your attention...